Modelling Multiple Discontinuities in Rectangular Waveguide Partially Filled with Non-Reciprocal Ferrites

Miguel A. Solano, Angel Vegas, and Andres Prieto

Abstract— **This work analyzes multiple 3-D discontinuities between rectangular ferrite-loaded waveguides. Each uniform section of waveguide is analyzed by means of Schelkunoff's method with improved convergence. Combining this method with the mode matching method allows the analysis of multiple discontinuities to be simplified. The theoretical results obtained in this way agree quite well with measurements.**

I. Introduction

THE STUDY OF non-reciprocal elements loaded with magnetized ferrites is very complex, unless the structures to be analyzed are limited to those containing slabs of ferrite magnetized in the transverse direction which fill the entire height of the waveguide [1], [2]. Complicated two-dimensional devices are usually analyzed by using simpler models with discontinuity in only one transverse direction; this is the case of the ferrite toroid phase shifter [3]. Rigorous analyses of this device have only been made by means of methods such as finite differences or coupled modes [4], [5]. Therefore, when analyzing discontinuities between waveguides with non-reciprocal media, it is common practice either to use simple models to characterize the problem or to consider discontinuities in only two dimensions, one transverse and the other in the longitudinal direction [6], [7].

Schelkunoff's method is a variational technique which has been successfully applied to both open and closed isotropic dielectric structures [8], [9]. The set of base modes that is normally employed (TE and TM modes of the empty waveguide) is complete for the expansion of the transverse components but not for the longitudinal ones. This fact, which does not have any influence on the study of dielectric structures, causes the method to present convergence problems when it is applied to the analysis of both isotropic and anisotropic magnetic structures. To overcome this problem it is necessary to introduce an additional term in the expansion of the H_z component [6]. This term corresponds to the solution of H_z for the TE modes, which is the result of letting $n = m = 0$. In this work Schelkunoff's method is generalized so as to be able to analyze ferrite-loaded waveguides with discontinuities in the two transverse directions and magnetization in any direction of the coordinate system.

The mode matching method has been used to analyze axial discontinuities between ferrite-loaded waveguides [1], [7]. The combination of this method with Schelkunoff's method eliminates the integrals involved in the mode matching method [8]; instead, they are calculated by means of a sum of products of coefficients provided by Schelkunoff's method. In this paper we propose an improvement of this combination of methods, which consists in using as the set of modes, on both sides of the discontinuity to be studied, the set of base modes employed in Schelkunoff's method instead of using the normal set of eigenmodes. This makes it possible to find the coefficients of the system of equations from which the scattering matrix is obtained by means of an appropriate reordering of the coefficients provided by Schelkunoff's method. This reduces drastically the complexity and the time employed in calculating the generalized scattering matrix of a discontinuity. Afterwards, the union of multiple discontinuities is performed by the normal procedure of linking generalized scattering matrices in non-reciprocal media [1].

II. Theory

A. Schelkunoff's Method

The transverse and longitudinal components of the electromagnetic fields of a cylindrical waveguide partially filled with dielectric or magnetic media which are not necessarily reciprocal (Fig. 1(a)) can be expressed as a linear combination of the base functions (TE and TM modes of the empty waveguide) [10]:

$$\vec{E}_t(x,y,z) = \sum_{i=1}^{N1} V_{(i)}(z)\vec{e}_{t(i)}(x,y) + \sum_{j=1}^{N2} V_{[j]}(z)\vec{e}_{t[j]}(x,y) \quad (1)$$

$$\vec{H}_t(x,y,z) = \sum_{i=1}^{N1} I_{(i)}(z)\vec{h}_{t(i)}(x,y) + \sum_{j=1}^{N2} I_{[j]}(z)\vec{h}_{t[j]}(x,y) \quad (2)$$

$$E_z(x,y,z) = \sum_{i=1}^{N1} V^z_{(i)}(z)e_{z(i)}(x,y) \quad (3)$$

$$H_z(x,y,z) = \sum_{j=1}^{N2} I^z_{[j]}(z)h_{z[j]}(x,y) + H_0(z) \quad (4)$$

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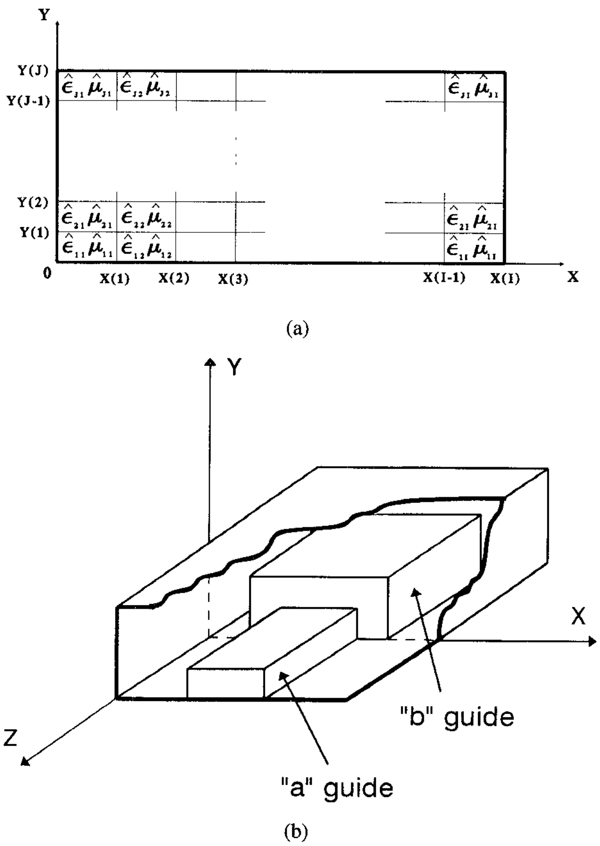


Fig. 1(a). Transverse section of a waveguide partially loaded with different anisotropic materials. (b) Discontinuity between two partially loaded waveguides.

In these equations the subindices within parentheses refer to the TM modes, the subindices within square brackets refer to the TE modes, and N1 and N2 are the number of TM and TE modes considered, respectively. The additional term H_0 introduced in expression (4), which is independent of the transverse coordinates, comes from the solution TE_{00} [6]. This term is null when non-magnetic structures are analyzed but is non-zero for magnetic structures.

Substituting these expressions into the Maxwell equations and then applying Galerkin's method leads to an equation of eigenvalues which, depending on the procedure utilized, may be of first [10] or second order [9]. The greater numerical stability of the first-order system allows solutions to be obtained even for waveguides with ferrites very close to resonance. The resulting expression for this case has the form

$$[Z][V,I] = \Gamma[V,I] \tag{5}$$

where the eigenvalues are the propagation constants and the eigenvectors are the coefficients of the equations (1) and (2). The expressions of the elements of the matrix (Z) are calculated in a way similar to that shown in [10] with the modification required by the introduction of the term H_0.

B. Mode Matching Method

This method consists in expressing the transverse electromagnetic field on each side of a discontinuity (Fig. 1(b)) as a combination of the modes of the corresponding waveguide and applying the continuity conditions in the interface [7], [8], that is to say

$$\sum_{i=1}^{M}(a_i\vec{e}_{ai} + a'_i\vec{e}\,'_{ai}) = \sum_{j=1}^{N}(b_j\vec{e}_{bj} + b'_j\vec{e}\,'_{bj}) \tag{6}$$

$$\sum_{i=1}^{M}(a_i\vec{h}_{ai} + a'_i\vec{h}'_{ai}) = \sum_{j=1}^{N}(b_j\vec{h}_{bj} + b'_j\vec{h}'_{bj}) \tag{7}$$

In these expressions, "M" and "N " are the number of eigenmodes selected in waveguides "a" and "b" respectively; a_i and b'_j are the coefficients of the modes that propagate toward the junction; and a'_i and b_j are the coefficients of the modes generated in the junction.

Next, a system of linear equations is generated by applying Galerkin's method and using the orthogonality relations which are satisfied in non-reciprocal media [11]. When this system of equations is resolved, the coefficients of each mode are found, resulting in the generalized scattering matrix of the discontinuity. An alternative to this process can be seen in [7] where discontinuities between one empty waveguide and another loaded with a ferrite slab magnetized transversally are analyzed. Since the modes in the ferrite-loaded waveguide do not satisfy the reciprocity theorem and, therefore, do not fulfill the classic orthogonality relations, in that work the eigenmodes of the anisotropic section were replaced by equivalent dielectric modes, i.e. the modes corresponding to a section of waveguide which is the same as the non-reciprocal waveguide but which has, in place of the ferrite, an isotropic dielectric of the same dimensions and permittivity. In this way, the problem could be treated as if it were an isotropic medium without losses. However, the procedure did not prove to be very efficient because the equivalent modes and the coefficients of the expansion had to be calculated by means of a moment method.

Along the same lines, in this work we have not taken the set of eigenmodes as base functions but, instead, we have used the set of base modes (the TE and TM modes of the empty waveguide) employed in Schelkunoff's method. The advantage of this procedure with respect to that used in [7] is that it is not necessary to calculate any new modes because they are the ones used in Schelkunoff's method, nor is it necessary to calculate its coefficients, since they are the eigenvectors that have already been calculated. This makes it possible to avoid the use of the orthogonality relations that hold for non-reciprocal media and replace them with the orthogonality properties of the modes of the empty waveguide. For this purpose, the eigenmodes on each side of the discontinuity can be expressed as a function of the base modes employed in Schelkunoff's method as is shown in the following expressions:

$$\left.\begin{aligned} \vec{e}_{ai} &= \sum_{j=1}^{NM} V_{ij}^{a}\vec{e}_j & \vec{h}_{ai} &= \sum_{j=1}^{NM} I_{ij}^{a}\vec{h}_j \\ \vec{e}\,'_{ai} &= \sum_{j=1}^{NM} V_{ij}'^{a}\vec{e}_j & \vec{h}'_{ai} &= \sum_{j=1}^{NM} I_{ij}'^{a}\vec{h}_j \end{aligned}\right\} \quad (i = 1, 2, \cdots, M) \tag{8}$$

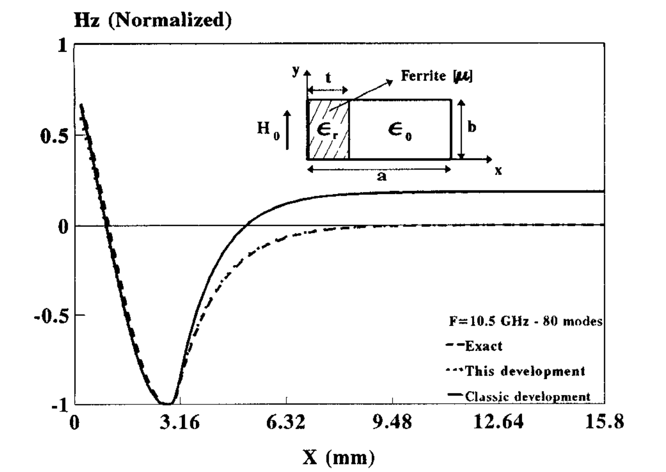


Fig. 2. Variation of the component H_z with the "x" coordinate for a slab of ferrite magnetized in the "y" direction. Dim. (mm.): $a = 15.8, b = 7.9, t = 3$. Characteristics of the ferrite: $\epsilon_r = 10, 4\pi M_s = 3000$ Gauss. External magnetic field $H_0 = 4000$ Oersteds.

$$\left.\begin{array}{ll}\vec{e}_{bk} = \sum_{j=1}^{NM} V_{kj}^{b}\vec{e}_j & \vec{h}_{bk} = \sum_{j=1}^{NM} I_{kj}^{b}\vec{h}_j \\ \vec{e}'_{bk} = \sum_{j=1}^{NM} V_{kj}^{\prime b}\vec{e}_j & \vec{h}'_{bk} = \sum_{j=1}^{NM} I_{kj}^{\prime b}\vec{h}_j\end{array}\right\} \quad (k = 1, 2, \cdots, N) \tag{9}$$

where V and I are the eigenvectors provided by Schelkunoff's method, and $\vec{e}_i$ and $\vec{h}_i\,(i = 1 \cdots NM)$ are the transverse electric and magnetic fields, respectively, of the modes TE and TM of the empty waveguide. Substituting these expressions into (6) and (7) and applying the orthogonality properties of the modes of the empty waveguide, we obtain the generalized scattering matrix of the individual discontinuity

$$[S] = ([B]^t)^{-1}[A]^t \tag{10}$$

where the matrices (A) and (B) are of the order $(M + N) \times (2NM)$ and are given by

$$[A] = \left[\begin{array}{c|c} [V_{ij}^{a}] & [I_{ij}^{a}] \\ \hline [-V_{kj}^{\prime b}] & [-I_{kj}^{\prime b}] \end{array}\right],$$

$$[B] = \left[\begin{array}{c|c} [-V_{ij}^{\prime a}] & [-I_{ij}^{\prime a}] \\ \hline [V_{kj}^{b}] & [I_{kj}^{b}] \end{array}\right] \tag{11}$$

As it can be seen, the matrices of the system of equations can now be found by simply reordering the eigenvectors obtained in Schelkunoff's method and, therefore, it is not necessary to perform any sort of numerical integration. For (11) to have a solution, the sum of the number of modes taken on each side of the discontinuity $(N + M)$ must be equal to twice the total number of base modes chosen $(2NM)$.

III. Results

A. Schelkunoff's Method

A.1. Full-Height Ferrite Slab: Introducing the term H_0 in (4) results in a marked variation in the values obtained for the component H_z with respect to those obtained with the classic approach [10]. Fig. 2 shows the variation in the component H_z (normalized to its maximum value) as a function of the "x" coordinate corresponding to the fundamental mode of the structure illustrated in the figure. With the values indicated in the Fig. 2, the ferrite slab is close to resonance. This structure was chosen because it can be resolved analytically in an exact way. Three lines are plotted: the first corresponds to the analytic solution, the second to the solution obtained by the classic procedure [10], and the third to the modal expansion of (4). It can be seen that when the parameter H_0 is not included, the longitudinal component of the magnetic field is not represented adequately. This erroneous description of the magnetic field also gives rise to an increase in the error in the calculation of the propagation constant, which becomes

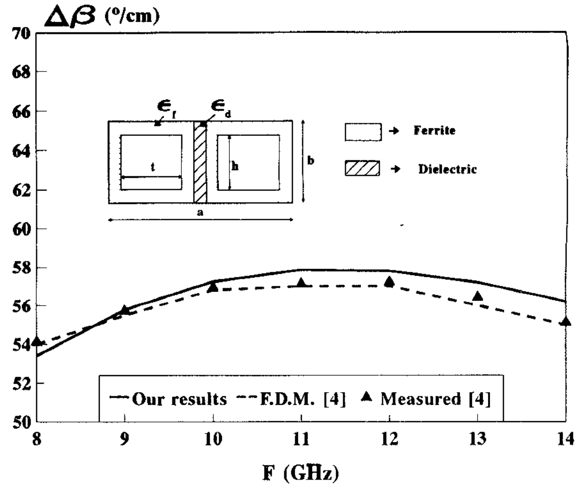


Fig. 3. Differential phase shift for a double ferrite toroid phase shifter with dielectric separation, as a function of the frequency. Dim. (mm.): $a = 13.4, b = 4, t = 4, h = 2$. Characteristics of the dielectric: $\epsilon_d = 24.96$. Characteristics of the ferrite YIG G-113: $\epsilon_f = 14.74, M_S(\mathrm{KA/m}) = 145, M_r(\mathrm{KA/m}) = 103$. Number of modes 30.

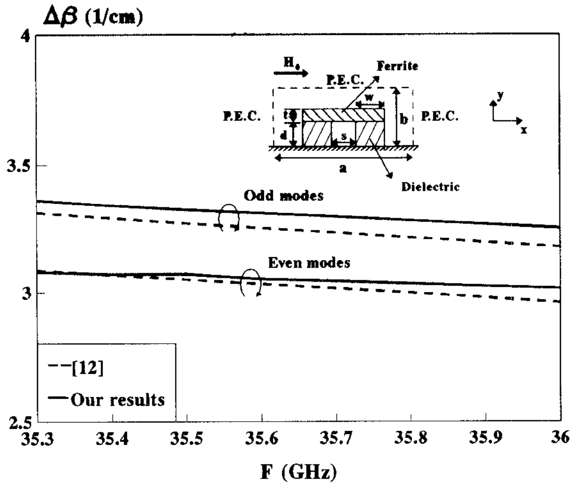


Fig. 4. Difference of phase between the even and odd, progressive and regressive modes as a function of the frequency. Dim. (mm): $a = 7, b = 3.5, w = 2.2, t = 0.35, d = 1.55, s = 0.2$. Dielectric: $\epsilon_r = 9.6$. Ferrite: $\epsilon_r = 12, 4\pi M_s = 1750$ Gauss. External magnetic field $H_0 = 6000$ Oersteds.

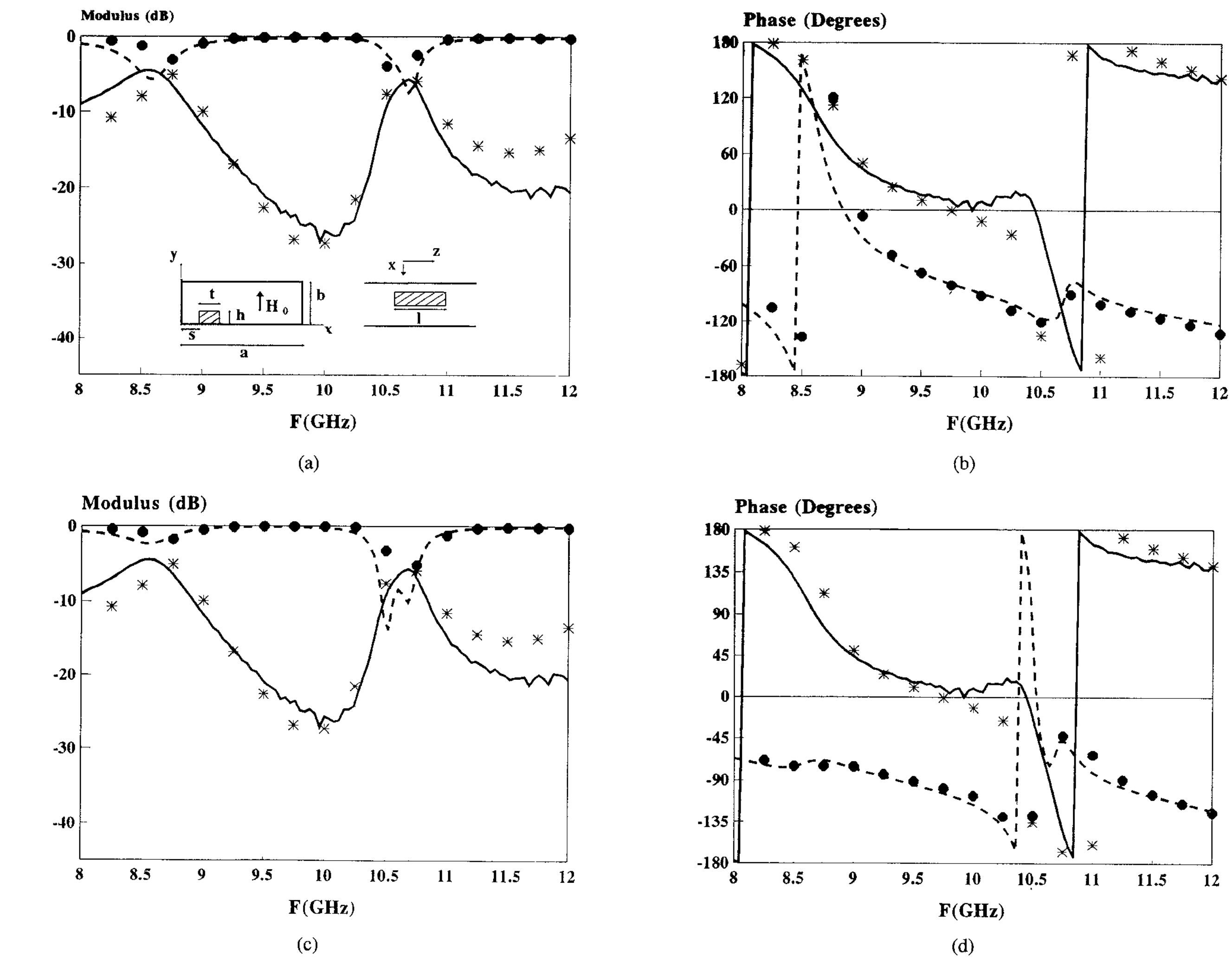


Fig. 5. Modulus (Fig. 5(a), (c)) and phase (Fig. 5(b), (d)) of the scattering parameters, as a function of the frequency, for two linked discontinuities. Dim. (mm.): $a = 22.86, b = 10.16, s = 0, t = 6, h = 3, l = 10.16$. Ferrite Y-10 from Trans. Tech. Inc. External magnetic field: $H_0 = 2000$ Oersteds. Number of modes 80. Fig. 5(a), (b). ({—}{—}) S_{11} and ({-} {-} {-} {-}) S_{21}: Measured. $(* * **) S_{11}$ and $(\bullet \bullet \bullet\bullet) S_{21}$: Theory. Fig. 5(c), (d). ({—}{—}) S_{22} and ({-} {-} {-} {-}) S_{12}: Measured. $(* * **) S_{22}$ and $(\bullet \bullet \bullet\bullet) S_{12}$: Theory.

progressively larger as the values of the components of the tensor increase, i.e. at frequencies close to resonance. Thus, when 80 basic modes are used, the error in the phase constant is 8.5% with the expansion of [10] while it is 2.4% with the expansion of (4).

A.2. Ferrite Toroid Phase Shifter: The ferrite toroid phase shifter is one of the structures with discontinuities in both transverse directions which has been most frequently modelled in order to obtain approximate solutions. The models that have been used are constructed by replacing the vertical parts of the toroid with vertical slabs of magnetized ferrite and the horizontal parts with air. Only recently has this structure been resolved without having to resort to approximate models. The rigorous analyses have used methods such as finite elements [4] or coupled modes, but with the classic expansion, [5]. For this structure, the difference between the development shown here and that used in [5] is very small, due to the fact that the piece of ferrite is magnetized in remanence and, therefore, the values of the components of the permeability tensor are close to one. Comparisons could not be made with the results in [5] because the published data do not provide sufficient detail about the characteristics of the materials used. Therefore, our results could only be compared with those published in [4]. Fig. 3 shows the theoretical and experimental results of [4] for the differential phase shift together with those we have obtained by expanding H_z according to (4). It can be seen that our results shows excellent agreement with the other ones, with the advantage that it requires much less simulation time than a finite element method. In addition, the same structure was analyzed without including the term H_0 and it was found that the differences with respect to the calculations including H_0 were in no case greater than a few tenths of a degree.

A.3. Coupler in Open Non-Reciprocal Waveguide: A non-reciprocal device of an open nature is the directional coupler constructed with two dielectric image guides, coupled by a sheet of ferrite that is transversally magnetized [12]. By producing a constructive and destructive interference between the odd and even, forward and reverse travelling modes, the coupler can be converted into an isolator. With Schelkunoff's method it is also possible to analyze the behavior of the surface modes of open structures. For this purpose it is necessary to enclose the open structure with perfectly electric conductor (PEC) and place them sufficiently far away that their presence will not have any influence on the behavior of the surface modes. The distance at which these metallic walls must be placed has been studied by various tests of convergence, and it was found that with an expansion of 80 base modes the influence of this shielding was negligible when its dimensions were 7×3.5 mm. Fig. 4 shows the variation of the difference of the phase constants between the even and odd, forward and reverse travelling modes, and compares them with the results obtained in [12] by using the effective dielectric constant method modified so as to take into account the anisotropy of the ferrite. The excellent agreement between our results and those of [12] can be observed. It was not possible to take into consideration the losses of the ferrite in the comparison because this parameter could not be taken into account in the theoretical method used in [12]. In addition, it was found that the working frequency of the resulting isolator is very sensitive to small variations in the magnitudes represented in Fig. 4.

B. Multiple Discontinuities

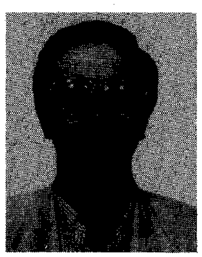
Our generalization of Schelkunoff's method has been checked mainly by means of comparisons with the results obtained by other authors. However, there is a lack of published results for 3-D discontinuities in waveguides containing ferrites, so that our theoretical results could only be compared with our own measurements. These measurements were carried out by means of a vectorial network analyzer in the frequency band 8–12 GHz using rectangular waveguide. The samples of ferrite were placed inside the rectangular waveguide and were held in place with pieces of foam. The static magnetic field was provided by an electromagnet whose poles produced a maximum value of about 3000 Oersteds, which could be considered uniform for the size of the ferrite pieces used. Fig. 5(a)–(d) show a comparison of the theoretical and experimental results for the scattering matrix obtained when two discontinuities were analyzed (empty waveguide—ferrite image waveguide—empty waveguide). The external magnetic field was 2000 Oersteds. Taking into account the demagnetization factors of the sample, a theoretical value of 1000 Oersteds was obtained for the internal field; this value was then used in the calculations. For this value of the magnetic field the ferrite was saturated. It can be seen that excellent agreement was obtained in the results for phase in the entire frequency band, whereas in the results for modulus there are slight discrepancies at the beginning and end of the band of frequencies analyzed. In the rest of the frequency band, the theory describes perfectly the actual performance of the structure. It can be seen that values of S_{11} and S_{22} are practically indentical, in agreement with the theoretical prediction [11], and that the value of the differential phase shift is almost constant at about 30°/cm in the central zone of the frequency band.

IV. Conclusions

Schelkunoff's method has been generalized to make it capable of analyzing three-dimensional dielectric and magnetic structures, and the convergence when applying it to magnetic structures has been improved. Combining this technique with the mode matching method greatly simplifies the calculations when studying multiple discontinuities between both reciprocal and non-reciprocal media. The combination of these two methods makes it possible to analyze and design non-reciprocal waveguide devices having 3–D discontinuities.

References

[1] J. Huer, F. Arndt and J. Bornemann, "Field theory design of ferrite loaded waveguide nonreciprocal phase shifters with multisection ferrite or dielectric slab impedance transformers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 6, pp. 552–560, June 1987.
[2] M. Koshiba and M. Suzuki, "Finite-element analysis of H-plane waveguide junction with arbitrary shaped ferrite post," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, no. 1, pp. 103–109, Jan. 1986.
[3] W. Hauth, "Accurate analysis of latching phase shifters," *Proc. Inst. Elec. Eng.*, vol. 133, no. 3, pp. 165–168, 1986.
[4] J. Svedin, B. Carlegrim and S. Hagelin, "Accurate design of ferrite toroid phase shifters," *18th Europ. Microwave Conf.*, pp. 391–396, Stockholm, 1988.
[5] Y. Xu and G. Zhang, "A rigorous method for computation of ferrite toroidal phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. 36, no. 6, pp. 929–983, Jun. 1988.
[6] H. Chaloupka, "A coupled-line model for the scattering by dielectric and ferrimagnetic obstacles in waveguides," *Arch. Elek. Übertragung*, vol. 34, pp. 145–151, Apr. 1980.
[7] F. J. Bernues and D. M. Bolle, "The ferrite-loaded waveguide discontinuity problem," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, no. 11, pp. 908–918, Nov. 1974.
[8] J. Rodríguez, M. A. Solano and A. Prieto, "Characterization of discontinuities in dielectric waveguides using Schelkunoff's method: Application to tapers and transitions," *Int. J. Electronics*, vol. 66, no. 5, pp. 807–820, 1989.
[9] K. Ogusu, "Numerical analysis of rectangular dielectric waveguide and its modifications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, no. 11, pp. 875–885, Nov. 1977.
[10] S. A. Schelkunoff, "Generalized telegraphist's equations for waveguides," *Bell Syst. Tech. J.*, vol. 31, pp. 784–801, July 1952.
[11] A. T. Villeneuve, "Orthogonality Relationships for Waveguides and Cavities with Inhomogeneous Anisotropic Media," *IRE Trans. Microwave Theory Tech.*, vol. MTT-7, pp. 441–446, Oct. 1959.
[12] S. W. Yun and T. Itoh, "Nonreciprocal wave propagation in hollow image guide with a ferrite layer," *Proc. Inst. Elec. Eng.*, vol. 132, Pt. H, no. 4, pp. 22–226, June 1985.



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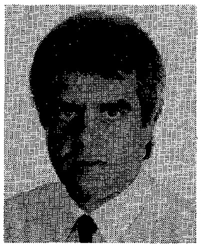

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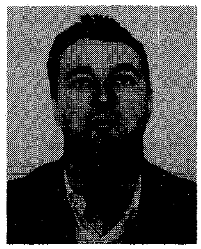

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